

Upper limit on the amplitude of gravitational waves around 0.1Hz from the Global Positioning System

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The global positioning system (GPS) is composed of thirty one satellites having atomic clocks with 10^{-15} accuracy on board and enables one to calibrate the primary standard for frequency on the ground. Using the fact that oscillators on the ground have been successfully stabilized with high accuracy by receiving radio waves emitted from the GPS satellites, we set a constraint on the strain amplitude of the gravitational wave background h_c . We find that the GPS has already placed a meaningful constraint, and the constraint on the continuous component of gravitational waves is given as $h_c < 4.8 \times 10^{-12} (1/f)$ at $10^{-2} \lesssim f \lesssim 10^0$ Hz, for stabilized oscillators with $\Delta\nu/\nu \simeq 10^{-12}$. Thanks to the advantage of the Doppler tracking method, seismic oscillations do not affect the current constraint. Constraints on h_c in the same frequency range from the velocity measurements by the lunar explorers in the Apollo mission are also derived.

I. INTRODUCTION

The existence of gravitational waves is predicted in general relativity and many modified gravity theories. By observing gravitational waves (GWs), we can not only test the general relativity and modified gravity theories in strong gravitational fields, but also reveal the merger process of compact objects through the history of the universe and physics in the early universe such as inflation (*e.g.* [1]). In particular, GWs which originate from the cosmological inflation have almost scale-invariant spectrum and propagate freely since their generation, and thus the detection of such scale-invariant GWs can be considered as a direct proof of the existence of cosmological inflation. In order to prove the primordial GWs to be scale-invariant, we have to observe GWs with a wide frequency range and high sensitivity.

Precise measurements of primordial GWs at high frequencies will tell us about the thermal history of the early universe, which could not be reached in other ways. For example, it has been suggested that the amplitude of GWs at the frequency range $f \gtrsim 10^{-2.5}$ Hz can be used to infer the reheating temperature [2], and the epoch of quark-gluon phase transition and neutrino decoupling at lower frequency range $10^{-10} \lesssim f \lesssim 10^{-8}$ Hz [3]. If the universe underwent a strongly decelerating phase after inflation realized in quintessential inflation models [4], inflationary GWs have a blue spectrum at higher frequency $f \gtrsim 10^{-2}$ Hz [5, 6]. Therefore, while the first detection of primordial GWs is expected through the CMB experiments which probe the waves at smallest frequencies (*e.g.* [7]) it is significant to explore gravitational waves with a wide range of frequency.

In addition, measurements of GWs at high frequency will open a new window on the black hole merger events in the universe. Matsubayashi *et al.* showed that the frequency of gravitational waves that originate from merg-

ers of intermediate mass black holes (IMBHs) falls inside the range of $10^{-5} \lesssim f \lesssim 10^1$ Hz, and one can discover or set constraints on these merger events within several gigapersec from the Earth in future gravitational wave observation projects [8].

In order to observe GWs, many kinds of gravitational wave detectors have been proposed and developed. The Laser Interferometer Gravitational Wave Observatory (LIGO) has already set a strong constraint on the strain amplitude $h_c < 4 \times 10^{-24}$ at the frequency of GWs around 10^2 Hz [9, 10]. However, seismic oscillations interfere ground-based observations of gravitational waves at lower frequencies as $f \lesssim 10$ Hz (*see* [11]). Aiming at this frequency range, Torsion bar detectors such as TOBA have been developed. Ishidoshiro *et al.* set a constraint on the strain amplitude of the continuous component of GWs as $h_c < 2 \times 10^{-9}$ at $f \sim 0.2$ Hz [12].

At slightly lower frequency range, with some planetary explorers, constraints such as $h_c < 1 \times 10^{-15}$ at $f = 10^{-2}$ Hz [13] and $h_c < 2 \times 10^{-15}$ at $f = 3 \times 10^{-4}$ Hz [14] have been set by the Doppler tracking method. However it is difficult to distinguish signals of GWs as the frequency of targeting GWs increases higher than 10^{-2} Hz due to the noise in the electric circuits of which are installed in the receiver in this method.

In this work, we consider the modulation of frequency of radio waves from GPS satellites by stationary/prompt GWs. GPS satellites emit precious, stable and high intensity radio waves in order to provide positional accuracy for the GPS in navigation. It is possible because the oscillators of GPS satellites are designed to be synchronized with atomic clocks which are loaded with the satellites. These oscillators are so stable that the fractional error of the frequency of their emitting radio waves is suppressed to $\Delta\nu/\nu < 10^{-15}$ [15].

The GPS method is an application of the Doppler tracking method which has two advantages. First, GPS method can probe in the frequency range $f \gtrsim 10^{-2}$ Hz

where the conventional Doppler tracking methods can not reach because GPS satellites are much closer than the planetary explorers, while the distance is large enough for the noise due to seismic oscillations to be negligible as shown in the next section. Secondly, the radio waves from GPS satellites can be detected everywhere and everytime on the ground. This condition will be suitable to make cross correlation study and crucial to detect GWs from prompt events.

In this study, we set a constraint on the amplitude of GWs for $0.01 \leq f \leq 1$ Hz by using the Doppler tracking method with GPS disciplined oscillators (GPSDOs). The GPSDO is the stable oscillator with a quartz oscillator whose output is controlled to agree with the signals from GPS satellites [15, 16]. To have GPSDOs operating with high stability, the amplitude of the continuous components of GWs should be small. Recently, the frequency stability of GPSDOs for a short time interval about from seconds to a few hundred seconds has been reached to the level of $\Delta\nu/\nu \simeq 10^{-12}$ [15, 16]. This short time stability of the frequency of GPSDOs enables us to set a constraint on the strain amplitude of the continuous GWs.

This paper is organized as follows. We estimate the effects of GWs on the measurements of the radio waves from GPS satellites and derive the constraint in §2. In our formulation, we adopt TT gauge to describe the GWs. In §3, we discuss implications of the result to the merger events of IMBHs and inflationary GWs. We conclude this paper in §4. Throughout this paper, ν and f mean the frequency of the electro-magnetic waves and the GWs, respectively. The speed of light is denoted by c . A dot represents a partial derivative respective to the physical time, *i.e.* $\dot{x} \equiv \partial x / \partial t$.

II. EFFECTS OF GWS ON GPS MEASUREMENTS

In this section, we consider the effect of gravitational wave backgrounds (GWBs) on the radio waves emitted by GPS satellites. Here we assume that GWBs are expected to be isotropic and stationary (*see.* [17]). For GWs whose wavelength are longer than the typical distance between GPS satellites and detectors on the ground, the frequency of radio waves emitted by the satellites is modulated as

$$\frac{\Delta\nu}{\nu} = \frac{l}{2c} \dot{h}(t) \sin^2 \theta, \quad (1)$$

where $\dot{h}(t)$, θ and l are the time derivative of the amplitude of GWs at time t , the angle between the directions of propagation of radio waves and gravitational waves, and the distance between the GPS satellite and the observer, respectively.

By assuming GWs are monotonic and plane waves parallel to the z -axis, the amplitude of GWs $h(t)$ and its time

derivative $\dot{h}(t)$ can be written with the strain h_c as

$$h(t) = h_c \sin \left(2\pi f \left(t - \frac{z}{c} \right) + \phi \right), \quad (2)$$

$$\dot{h}(t) = 2\pi f h_c \cos \left(2\pi f \left(t - \frac{z}{c} \right) + \phi \right), \quad (3)$$

where ϕ is the phase of GWs at $z = t = 0$.

From Eq. (1), signals of GWs generate an additional shift of frequency of radio waves. When one considers the case where $\theta = \pi/2$, the effect of GWs on the frequency modulation of electro-magnetic waves can be estimated as

$$\frac{\Delta\nu}{\nu} = \frac{\pi f l}{c} h_c. \quad (4)$$

In reality, the signal sourced by GWs is buried within the noise. Therefore, if one can receive the radio waves which is emitted at a distance of l with a time variance of the frequency fluctuations σ [18], one can set the upper bound of the strain amplitude of gravitational waves as,

$$h_c < \frac{c}{\pi l f} \sigma. \quad (5)$$

For the GPS, the distance l is approximately 2×10^7 m and the standard variation of frequency from GPS satellites converges to $\sigma \simeq 1 \times 10^{-12}$ by integrating the signal from GPS satellites for the period from one second to one hundred seconds [15, 16]. By receiving the signal from the GPS for a period t_i , the frequency range of the GWs which one can probe is limited as $f \geq t_i^{-1}$. By combining it with the condition that the wavelength of GWs is longer than the distance between the GPS satellites and the observer, the probed range of frequency with the GPS can be written as

$$t_i^{-1} \leq f \leq \frac{l}{c}. \quad (6)$$

From Eq. (1) because the amplitude of the modulation is proportional to the frequency of GWs f , the strain amplitude h_c is constrained tighter for higher frequency. By substituting the numbers into Eq. (5), we can set a constraint on the strain of GWBs as

$$h_c < 4.8 \times 10^{-12} \left(\frac{1\text{Hz}}{f} \right). \quad (7)$$

In figure 1, we plot the constraint on the strain amplitude of the continuous component of GWs and compare the result from the torsion bar detector [12]. We find that the GPS gives a tighter constraint on the relevant frequency range.

The GPS method does not suffer seismic oscillations that disturb ground-based observations. The reason can be given as follows. In the GPS method, the effect of GWs can be seen as changes of observed distances between the satellites and observers. When plane and monotonic GWs come to an observer, the observed distance l between the satellite and the observer on the

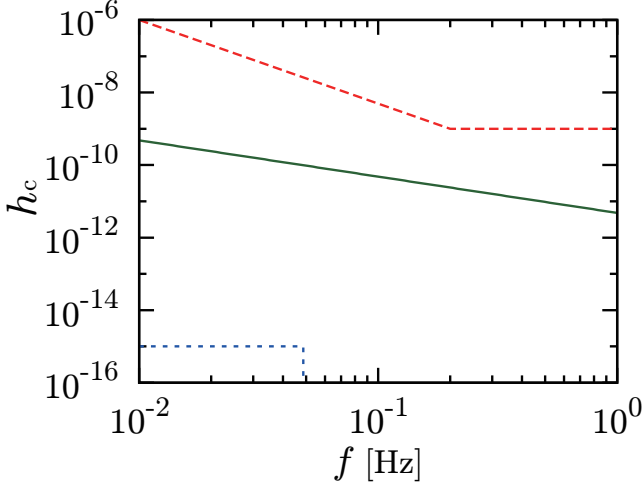


FIG. 1: The upper limit on the strain amplitude of GWs from GPS satellites (solid line). The dashed line represents the constraint from the torsion bar detector [12]. The dotted line represents the constraint from the Doppler tracking method [13].

ground can be written in terms of the proper distance l_0 as

$$l = l_0 (1 + h) + x_s, \quad (8)$$

where h is the amplitude of GWs and x_s is the amplitude of seismic oscillations. Thus the effect of seismic oscillations relative to the strain amplitude of GWs can be characterized by x_s/l_0 . Shoemaker *et al.* reported a typical power spectrum of seismic oscillations as [19]

$$x_s \simeq 3 \times 10^{-7} (f/1 \text{ Hz})^{-2} \text{ [m]}. \quad (9)$$

Because $l_0 \simeq 2.0 \times 10^7$ m, the effect of seismic oscillations is suppressed as,

$$\frac{x_s}{l_0} \simeq 1.5 \times 10^{-14} (f/1 \text{ Hz})^{-2}. \quad (10)$$

This is smaller than the upper limit of our constraint, Eq (7).

In addition, because GPS satellites fly much closer to the ground than the planetary explorers which have been used in the Doppler tracking method, we can set a constraint on the strain amplitude of GWs in the higher frequency region than those of ULYSSES and Cassini ([13, 14]).

The intensity of GWs can be characterized by the dimensionless cosmological density parameter $\Omega_{\text{gw}}(f)$. The parameter is defined as

$$\Omega_{\text{gw}}(f) = \frac{10\pi^2}{3H_0^2} (fh_c)^2, \quad (11)$$

where H_0 is the Hubble constant, and the Planck collaboration reported as $H_0 = 67.11 \text{ km/sec/Mpc} = 2.208 \times$

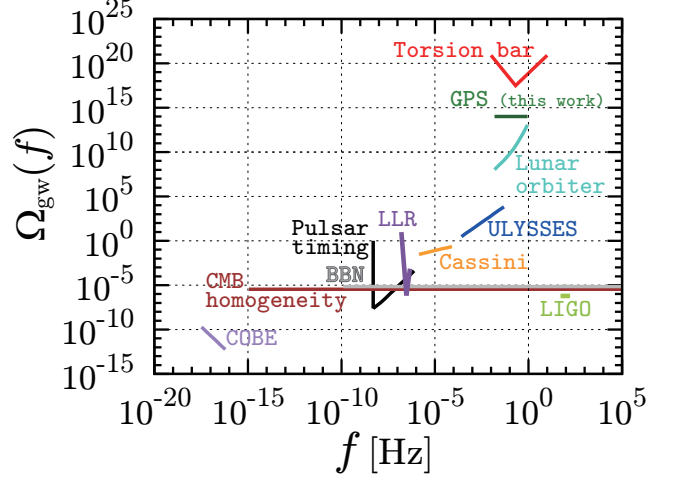


FIG. 2: Summary of the constraints on GW background in terms of $\Omega_{\text{gw}}(f)$, which includes COBE [17], CMB homogeneity and BBN [21], Pulsar timing [22], LLR [23, 24], Cassini [14], ULYSSES [13], Lunar orbiter [25], Torsion bar [12], LIGO [9, 10], and GPS.

$10^{-18} \text{ sec}^{-1}$ [20]. Then the constraint Eq. (7) can be written as

$$\Omega_{\text{gw}}(f) < 1.7 \times 10^{14} \text{ for } 10^{-2} \lesssim f \lesssim 10^0 \text{ [Hz]}. \quad (12)$$

In figure 2, we compare our constraint with the previous ones. It can be seen that the Doppler tracking method with the GPS is setting a constraint on the amplitude of continuous component of gravitational waves in the frequency range of $10^{-2} \lesssim f \lesssim 10^0$ Hz, a window between the constraints from the Torsion bar and the planetary explores.

III. DISCUSSION

In this work, we show that satellites with atomic clocks are available for setting constraints on the strain amplitude of GWs at $10^{-2} \lesssim f \lesssim 10^0$ Hz. The constraints based on the Doppler tracking method with planetary explorer such as ULYSSES and Cassini are mainly limited from the stability of the hydrogen maser clock on the ground $\sim 10^{-15}$ at the frequency range $10^{-6} \lesssim f \lesssim 10^{-2}$ Hz. The stability of the optical lattice clock is expected to reach 10^{-18} [26]. If future explorers have the optical lattice clock on board, they will become useful instruments for detection/setting constraints on the strain amplitude of GWs (*see also* [27]).

Determination accuracy of frequency fluctuations of received radio waves from GPS satellites depends mostly on the time resolution of the received electric-signal in the A/D converter of the GPS receiver σ_r . It is reported that $\sigma_r \geq 2 \times 10^{-13}/t_i$ and the largest error comes from the frequency transfer [28]. It is difficult to improve the

GPS constraints on the strain amplitude until the resolution of the quantization in the A/D converter is much improved.

Here let us apply our result to setting a constraint on the merger events of compact objects. In particular, it is predicted that mergers of the binary of IMBHs emit large GWs in the relevant frequency range, while it is difficult to detect the events through X-rays or radio waves if the systems do not have accretion disks. Therefore observations or constraints of GWs enable one to estimate the number density of the binary of IMBHs and the merger events.

During the quasi-normal (QNM) phase of the IMBHs mergers, in which merging IMBHs are expected to emit the largest amplitude of GWs, the amplitude h_{QNM} , typical frequency f_{QNM} and the duration time of the event t_{QNM} are given by [29],

$$f_{\text{QNM}} \simeq 4 \times 10^{-2} \left(\frac{M}{10^6 M_\odot} \right)^{-1} [\text{Hz}], \quad (13)$$

$$h_{\text{QNM}} \simeq 2 \times 10^{-12} \left(\frac{M}{10^6 M_\odot} \right) \left(\frac{\varepsilon}{10^{-2}} \right)^{\frac{1}{2}} \left(\frac{R}{4 \text{ kpc}} \right)^{-1} \quad (14)$$

$$t_{\text{QNM}} \simeq 30 \times \left(\frac{M}{10^6 M_\odot} \right) [\text{sec}]. \quad (15)$$

Here ε is the eccentricity of the orbit, R is the distance to the binary, and we assumed that two IMBHs have the same mass M . From Eqs. (7), (13) and (14), one can rule out merger events of IMBHs from the GPS constraint as

$$R \gtrsim 0.1 \left(\frac{\varepsilon}{0.01} \right)^{-1/2} [\text{kpc}] \quad (16)$$

for $4 \times 10^4 M_\odot \leq M \leq 4 \times 10^6 M_\odot$.

The minimum and maximum masses are determined from the frequency range we can probe in this method, i.e. Eq.(4).

Frequency modulation signals induced by GWs may be disturbed by the plasma effect in the ionosphere and the atmosphere. Fluctuations of the column density of free electrons in the plasma, called the dispersion measure (DM), induce those in frequency of GPS radio waves as (*e.g.* [30, 31])

$$\frac{\Delta\nu}{\nu} = \frac{e^2 \nu^{-2}}{2\pi m_e c r} \text{DM}, \quad (17)$$

where $\text{DM} = \int_0^r n_e ds$ is the column density of electrons, m_e and e are the mass and the electric charge of an electron, respectively, and r is the distance between the GPS satellite and the observer. By comparing the above equation with Eq. (4), one can see that the frequency dependences are different between the modulations induced by GWs and the plasma effect. Therefore the modulation originated from the plasma can in principle be removed by using multiple frequencies. Some of the GPSDOs observe the two bands of GPS radio waves (L1 & L2) [39] and take into account this modulation.

However, even when GPSDOs give the best performance, the distance at which one can detect the merger of IMBHs with GPSDOs is limited only to 0.1 kiloparsec from the Earth. In order to detect mergers of IMBHs and compact objects and the GW background from inflation with realistic amplitude, space-based gravitational wave detectors are needed. As future gravitational wave detectors, Laser Interferometer Space Antenna (LISA) and Deci-hertz Interferometer Gravitational wave Observatory (DECIGO) are in progress. LISA and DECIGO are expected to reach $h_c \simeq 3 \times 10^{-21}$ at $f \simeq 6 \times 10^{-3}$ Hz and $h_c \simeq 2 \times 10^{-24}$ at $f \simeq 0.3$ Hz, respectively [32, 33]. These sensitivities will enable us to detect almost all the merger events of IMBHs with mass $M \sim 10^3 M_\odot$ within the current horizon [40], and to reveal the properties of the strong gravity field and the cosmological inflation [29].

Finally let us discuss another constraint that can be obtained from the lunar orbiting explorers in a similar way to the GPS constraint. In the Apollo mission, in order to study the gravity field of the moon, lunar explorers such as Apollo 15 and 16 measured the change in distance between the explorers and the Earth precisely via S-band transponders [25]. The typical distance between the explorers and the ground is $l \approx 3.8 \times 10^8$ m, which is much longer than that of the GPS case. It was reported that anomalous oscillating motions had never been found in their every ten-second sampling data with 1×10^{-4} m/sec accuracy [25]. From Eqs. (3) and (8), this result is translated to the strain amplitude of GWs as $h_c < 2.6 \times 10^{-13}$ at the frequency range $f < c/l \simeq 0.8$ Hz [41]. The constraint is also depicted in Fig. 2. Recent lunar explorers such as Kaguya [34] and GRAIL [35, 36] may improve this upper bound. In addition, future measurements with lunar surface transponders such as those proposed by Gregnanin *et al.* [37] will be available for setting constraints on h_c .

IV. CONCLUSION

We set a constraint on the strain amplitude of the continuous component of GWs as $h_c < 4.8 \times 10^{-12} (1[\text{Hz}]/f)$ at the frequency range $10^{-2} \lesssim f \lesssim 1$ Hz with the radio waves emitted by GPS satellites in operation. Because the distance between GPS satellites and observers is order 10^7 m, seismic oscillations do not affect the constraints on the strain amplitude. The sensitivity to the GWs is limited to that of the A/D converter on the GPS receiver at the frequency range.

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 - [40] The current horizon scale is approximately equal to the distance to the last scattering surface of CMB $d_{A(\text{CMB})}$. Jarosik *et al.* reports that $d_A = 14.116^{+0.160}_{-0.163}$ Gpc [38].
 - [41] To be more precise, the distance between the moon and the earth is so long that this constraint should be

corrected at frequencies above 0.1Hz as $h_c < 2.6 \times 10^{-13} \sqrt{1 + (f/0.16 \text{ [Hz]})^2}$. At $f = 1 \text{ Hz}$, we obtain the

constraint $h_c < 1.6 \times 10^{-12}$.